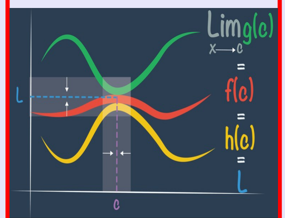
Math 261

Fall 2022

Lecture 29



Given $y = f(x)$

Critical numbers $\Rightarrow$ $f'(x) = 0$ or $f'(x)$ undefined

Critical points $\Rightarrow$ Evaluate $f(x)$ at C.N.

Possible Inflection Points $\Rightarrow$ $f''(x) = 0$ or $f''(x)$ undefined

Inflection points are those points where Concavity changes $\Rightarrow$ $f''(x) > 0 \Rightarrow$ C.U.

$f''(x) < 0 \Rightarrow$ C.D.

$f(x) = x^2 - 6x + 5$

$f'(x) = 2x - 6$

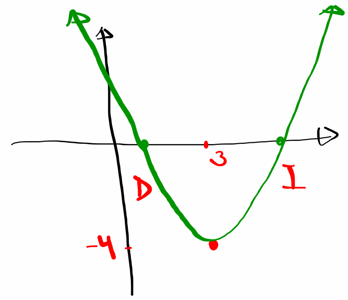
C.N. $\Rightarrow f'(x) = 0 \Rightarrow 2x - 6 = 0 \quad x = 3$

Critical point $(3, f(3)) = (3, -4)$

$f''(x) = 2 \qquad f''(x) > 0 \Rightarrow$ Concave Up

x	$-\infty$	3	∞
$f'(x)$	$-$	•	$+$
$f''(x)$	$+$		$+$
$f(x)$	↘		↗

Min. Point $(3, -4)$



$f(x) = \sqrt{x}$

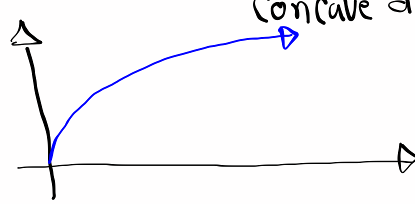
Domain $[0, \infty)$

$f'(x) = \dfrac{1}{2\sqrt{x}} \qquad f'(x) > 0 \qquad f(x)$ is increasing

$f'(x) = \frac{1}{2} x^{-1/2}$

$f''(x) = \frac{1}{2} \cdot \frac{-1}{2} x^{-3/2} = \dfrac{-1}{4x^{3/2}} = \dfrac{-1}{4x\sqrt{x}} \qquad f''(x) < 0 \qquad f(x)$ is concave down

x	0	∞
$f'(x)$	$+$	
$f''(x)$	$-$	
$f(x)$	↗	



$f(x) = x^4 - 4x^3$

Polynomial $\rightarrow$ Cont. everywhere $\rightarrow$ Domain $(-\infty, \infty)$

$f'(x) = 4x^3 - 12x^2 = 4x^2(x-3)$

C.N. $x = 0$ $x = 3$

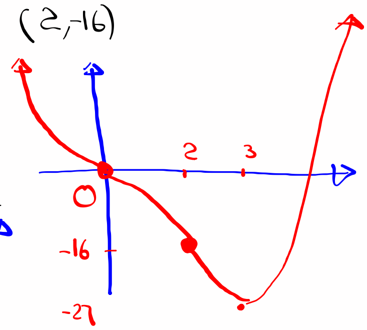
C.P. $(0, 0)$ $(3, -27)$

$f''(x) = 12x^2 - 24x = 12x(x-2)$

P.I.P. $x=0$ $x=2$

$(0,0)$ $(2,-16)$

x	$-\infty$ to 0	0 to 2	2 to 3	3 to ∞
$f'(x)$	$-$	$-$	$-$	$+$
$f''(x)$	$+$	$-$	$+$	$+$
$f(x)$	↘	↘	↘	↗



$f(x) = x^{2/3}(6-x)^{1/3}$ $f'(x) = \dfrac{4-x}{x^{1/3}(6-x)^{2/3}}$ $f''(x) = \dfrac{-8}{x^{4/3}(6-x)^{5/3}}$

Domain: $(-\infty, \infty)$

x-Int $(0,0), (6,0)$

Y-Int $(0,0)$

C.N.

$f'(x) = 0 \rightarrow x = 4$

$f'(x)$ undefined $\rightarrow x=0, x=6$

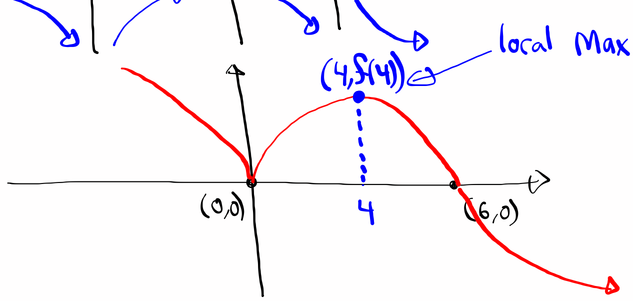
P.I.P.

$f''(x) \neq 0$

$f''(x)$ undefind

$x=0, x=6$

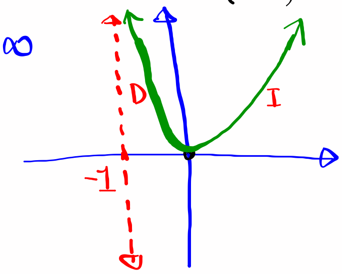
x	$-\infty$ to 0	0 to 4	4 to 6	6 to ∞
$f'(x)$	$-$	$+$	$-$	$-$
$f''(x)$	$-$	$-$	$-$	$+$
$f(x)$	↘	↗	↘	↘



$f(x)=\dfrac{x^2}{\sqrt{x+1}}$ $\qquad f'(x)=\dfrac{x(3x+4)}{2(x+1)^{3/2}}$ $\qquad f''(x)=\dfrac{3x^2+8x+8}{4(x+1)^{5/2}}$

Domain $(-1,\infty)$ $\qquad \lim\limits_{x\to -1^+} f(x)=\infty$

V.A. at $x=-1$



$\lim\limits_{x\to\infty} f(x)=\infty$ $\qquad$ x-Int $(0,0)$

Y-Int $(0,0)$

$f'(x)=\dfrac{x(3x+4)}{2(x+1)^{3/2}}$ $\qquad f'(x)=0$

$x=0,\ \cancel{x=\frac{-4}{3}}$ (Not in the domain)

$f''(x)=\dfrac{3x^2+8x+8}{4(x+1)^{5/2}}>0 \to$ C.U.

$3x^2+8x+8=0$

$a=3 \quad b=8 \quad c=8$

$x=\dfrac{-8\pm\sqrt{8^2-4(3)(8)}}{2(3)}=\dfrac{-8\pm\sqrt{-}}{6}$ No real Soln.

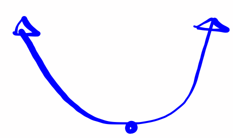
Find two numbers with differece of 100 and minimum product. $\to x$ & y

$x-y=100$

$x-100=y$

Product $=xy$

Product $=x(x-100)$

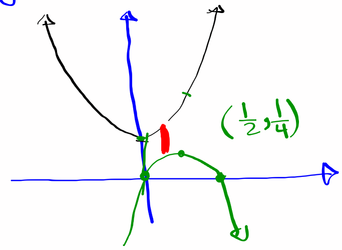


$P(x)=x^2-100x$ $\qquad$ Min. $(50,-50)$

$P'(x)=2x-100$ $\qquad x=50$

$P''(x)=2>0 \to$ C.U. $\to$ Min. Value

Graph $y = x^2 + 1$ & $y = x - x^2$ in the Same Coordinate System.



$y = x - x^2$

x-Int $(0,0), (1,0)$

$y' = 1 - 2x$

$y' = 0 \rightarrow x = \frac{1}{2}$ $(\frac{1}{2}, \frac{1}{4})$

Find the minimum vertical distance.

Vertical distance = Top − Bottom

$= x^2 + 1 - (x - x^2)$

$= x^2 + 1 - x + x^2$

$\rightarrow \boxed{D(x) = 2x^2 - x + 1}$

$D'(x) = 4x - 1$

$D'(x) = 0$

$x = \frac{1}{4}$

$D''(x) = 4 > 0$

C.U.

Min. point

$(\frac{1}{4}, \quad)$

$D(\frac{1}{4}) = 2(\frac{1}{4})^2 - \frac{1}{4} + 1$

$= 2 \cdot \frac{1}{16} - \frac{1}{4} + 1$

$= \frac{1}{8} - \frac{1}{4} + 1$

$= -\frac{1}{8} + 1 = \boxed{\frac{7}{8}}$